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Long-range Coulomb forces in DIS: missed radiative corrections?

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Abstract. The Born approximation, one photon exchange, used for DIS (deep inelastic scattering) is subject to virtual radiative corrections which are related to the long-range Coulomb forces. They may be sizeable for heavy nuclei since $Z\alpha$ is not a small parameter. So far, these corrections are known only for two processes, elastic scattering and bremsstrahlung on the Coulomb field of a point-like target. While the former amplitude acquires only a phase, in the latter case also the cross-section is modified. Although the problem of Coulomb corrections for DIS on nuclei is extremely difficult, it should be challenged rather than "swept under the carpet". The importance of these radiative corrections is questioned in the present paper. We show that, in the simplest case of a constant hadronic current, the Coulomb corrections provide a phase to the Born amplitude, therefore the cross-section remains the same. Inclusion of more realistic hadronic dynamics changes this conclusion. The example of coherent production of vector mesons off nuclei reveals large effects. So far a little progress has been made deriving lepton wave functions in the Coulomb field of an extended target. Employing available results based on the first-order approximation in $Z\alpha$, we conclude that the Coulomb corrections are still important for heavy nuclei. We also consider an alternative approach for extended nuclear targets, the eikonal approximation, which we demonstrate to reproduce the known exact results for Coulomb corrections. Calculating electroproduction of vector mesons, we again arrive at a large deviation from the Born approximation. We conclude that one should accept with caution the experimental results for nuclear effects in DIS based on analyses done in the Born approximation.

PACS. 25.30.-c Lepton-induced reactions – 25.30.Rw Electroproduction reactions – 13.40.-f Electromagnetic processes and properties – 25.30.Bf Elastic electron scattering

1 Introduction

Smallness of the fine structure constant α usually justifies lowest-order perturbative QED calculations. In some cases, however, the expansion parameter is not small, for instance, for interactions with heavy nuclei, where $Z\alpha \sim 1$. Therefore, the validity of Born approximation for deep inelastic scattering (DIS) should be questioned since the incoming and outgoing leptons propagate in the Coulomb field of the target, as is illustrated in fig. 1, which can cause a deviation of the DIS cross-section from the Born form. This may lead to important modifications of experimental data for nuclear effects in DIS which rely on analyses based on the Born approximation.

The simplest example is the elastic lepton scattering in the Coulomb field of a nucleus. Neglecting the nuclear structure, *i.e.* treating the target as a point-like charge, one can solve this problem exactly. Gordon [1] and Mott [2] have done it in the framework of nonrelativistic quantum mechanics. It turns out that the resulting amplitude is different from the Born approximation only by a phase which has opposite signs for positive and negative leptons. Thus, the elastic cross-section has no Coulomb corrections. On the other hand, in the case of hadronic elastic scattering the Coulomb effects do modify the cross-section because of the relative Coulomb phase [3–5].

In the relativistic case the problem of elastic Coulomb scattering was solved for spinless particles by Schrödinger [6] and for fermions by Mott [2] and Darwin [7]. Although the scattering phases were calculated, summation of the partial amplitudes, *i.e.* construction of the wave function of a relativistic charged particle in the Coulomb field, is still a challenge.

Nevertheless, for practical applications one can employ the approximation of high orbital momenta, $L^2 \gg (Z\alpha)^2$, which helps to solve the problem. Corresponding wave functions in the Coulomb field were found by Furry [8] and Maue and Sommerfeld [9].

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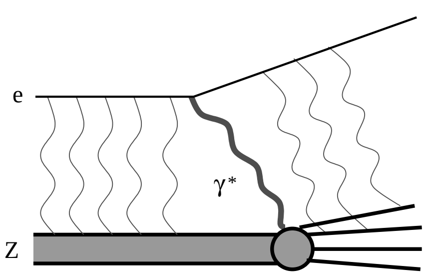

Fig. 1. Deep inelastic electron-nucleus scattering. The thin wavy lines illustrate the long-range Coulomb forces which violate the DIS kinematics ($\vec{q} \neq \vec{p}_1 - \vec{p}_2$) and modify the lepton wave functions.

Bethe and Maximon [10] made use of the same approximation to solve a more complicated problem of bremsstrahlung and production of electron-positron pairs by an electron scattering in the Coulomb field. Their finding is interesting, the long-range Coulomb forces modify not only the phase, but lead to a suppression of the cross-section compared to the Born approximation. The effect is significant, about 10–15 % for heavy nuclei. The fact that the Coulomb forces suppress radiation can be intuitively understood as a manifestation of the Landau-Pomeranchuk effect. Indeed, the electron trajectory is bending smoothly over long distances, and the photons radiated from different parts of the electron path interfere destructively.

One may expect similar modifications in other reactions, especially those which involve strong interactions, like DIS, they should be revisited aiming to clarify the role of long-range Coulomb forces. All analyses of DIS data are based on the Born approximation for the electromagnetic interaction with the target, *i.e.* the wave functions of the upcoming and outgoing leptons are treated as plane waves. At the same time, precise measurements [11] disclose rather fine, few percent nuclear effects, like small antishadowing at $x \sim 0.1$, or variation of nuclear shadowing with Q^2. It is still questionable how these small effects are related to the missed distortions generated by the Coulomb field.

On the other hand, the HERMES experiment [12] has discovered recently unusually strong nuclear effects which cannot be understood within conventional approaches [13]. Since these effects arise only after the radiative corrections are introduced into the analysis, it also motivated us to have a closer look at the problem of reliability of radiative corrections.

This paper is organized as follows. In sect. 2 we start with a specific simplified example of DIS with a hadronic current independent of Q^2. In this case the amplitude is proven to gain only a phase leaving the cross-section unmodified. This is a new nontrivial result which we utilize for further applications in this paper.

As a more realistic, but still simple example we choose the coherent vector meson production off nuclei which is treated within the vector dominance model (VDM). In sect. 3.1, we consider the case of a point-like target and

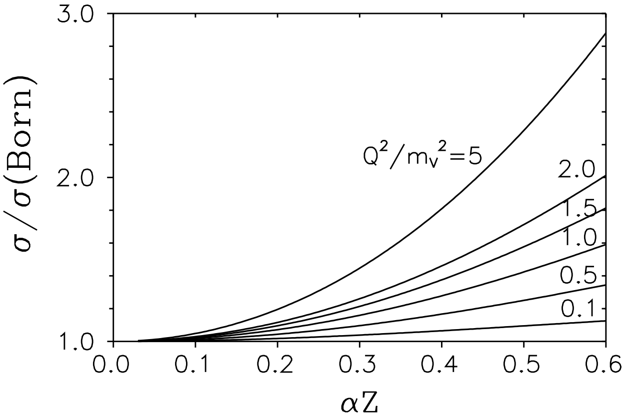

Fig. 2. Deviation of the cross-section from the Born approximation as a function of $Z\alpha$ for different fixed values of Q^2/m_V^2.

arrive at a substantially modified cross-section which deviates from the Born approximation. The Coulomb corrections increase with Q^2 as is shown in fig. 2.

The next step toward a realistic hadronic current is done in sect. 3.2. It corresponds to the vector meson production off an extended nucleus, but with the same leptonic wave functions calculated for a point-like Coulomb center. Again large corrections to the cross-section are found, whose value and sign varies with lepton energy and Q^2 as is demonstrated in fig. 3.

Eventually, in sect. 3.3, we replace the exact wave functions of the lepton in the Coulomb field of a point-like charge by approximate ones calculated for the case of an extended nuclear target. Unfortunately, the only solution available in the literature corresponds to the first-order expansion in $Z\alpha$. Assuming that it is not a phase, we arrive at a significant correction up to 13% for heavy nuclei.

In sect. 4 we try a different technique, the eikonal approximation, which is designed in a way which allows to perform calculations for nuclei of large size. First of all, we check whether the eikonal approximation is able to reproduce the Coulomb corrections previously calculated exactly. We demonstrate in Appendix B for the case of e^+e^- production off nuclei that the exact results for the Coulomb corrections are fully reproduced by the eikonal approximation. We calculate the Coulomb corrections for the process of vector meson production in the eikonal approximation in the limit of very high energies. Again the corrections are found to be large and to saturate as functions of Q^2 at $Q^2 > 0.1\,\text{GeV}^2$, as is shown in fig. 4.

Although our results are too rough to be incorporated into analyses of data for DIS, we conclude in sect. 5 that one should accept with a precaution the available experimental results for nuclear effect in DIS. We have found only one specific case of an oversimplified hadronic current when the Coulomb corrections have the form of a phase. In more realistic situations, the hadronic current consists of a few (or many) terms which acquire different phases and lead to a modified cross-section. Thus, all available data for DIS on heavy nuclei based on analyses utilizing the Born approximation might change if the Coulomb corrections are applied.

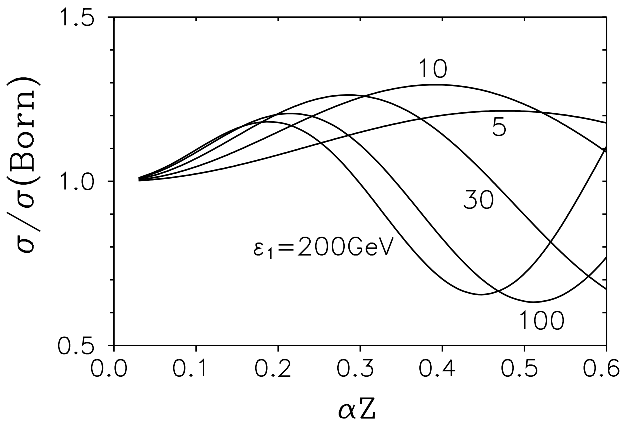

Fig. 3. The same as in fig. 2 but for different incident energies $\epsilon_1 = 200, 100, 30, 10$ and $5\,\mathrm{GeV}$ at fixed $y = 0.6$ and $Q^2 = m_\rho^2$.

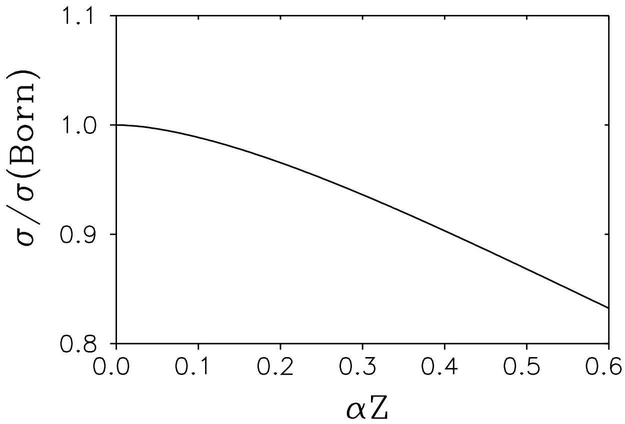

Fig. 4. The same as in fig. 2, but calculated in the eikonal approximation. The curve is calculated at $Q^2 > 1\ \mathrm{GeV}^2$ and is independent of Q^2 within the range of coherence $q_L \lesssim 1/R_A$.

2 A little theorem

This section is aimed to demonstrate that, for a specific simple form of the conserved hadronic current, the effects of long-range Coulomb forces are reduced to a phase factor, like it happens for elastic scattering. This result then will be implemented into more complicated situations.

Let us start with a process which leaves the nucleus intact, for example coherent electroproduction of vector mesons. The amplitude of this reaction in Born approximation reads

$$M(eA \to e'VA) \propto \int \mathrm{d}^4x\mathrm{d}^4y \langle p_2|j_\mu(x)|p_1\rangle D_{\mu\nu}(x-y)\langle V|J_\mu(y)|0\rangle\,, \quad (1)$$

where

$$\langle p_2|j_\mu(x)|p_1\rangle = \bar{\Psi}_{p_2}(x)\,\gamma_\mu\,\Psi_{p_1}(x) \quad (2)$$

is the operator of lepton current; $J_\mu(y)$ is the hadronic current operator; $D_{\mu\nu}(x-y)$ is the Green function of the photon. Using the Feynman gauge we have

$$D_{\mu\nu}(x-y) = -g_{\mu\nu}\int \mathrm{d}^4Q\frac{e^{iQ(x-y)}}{Q^2+io}\,, \quad (3)$$

and

$$M(eA\to e'VA)\propto -\int\frac{\mathrm{d}^4Q}{Q^2+io}j_\mu(Q;p_1,p_2)J_\mu(Q;p_V)\,. \quad (4)$$

Here the Fourier transform of the currents reads:

$$j_\mu(Q;p_1,p_2) = \int \mathrm{d}^4x\langle p_2|j_\mu(x)|p_1\rangle e^{iQx}\,; \quad (5)$$

$$J_\mu(Q;p_V) = \int \mathrm{d}^4y\langle p_V|J_\mu(y)|0\rangle e^{-iQy}\,; \quad (6)$$

$p_{1,2}$ are the initial and final lepton momenta; P_V is the momentum of the produced V.

If the initial and final wave functions of the lepton are undistorted plane waves,

$$\Psi_{p_1,p_2}(x) = e^{ip_{1,2}x}\,u(p_{1,2})\,, \quad (7)$$

then

$$j_\mu(Q;p_1,p_2) = (2\pi)^4\,\delta(p_1-p_2-Q)\,\bar{u}(p_2)\gamma_\mu u(p_1)\,, \quad (8)$$

and

$$M(eA\to e'VA) \propto \frac{1}{(p_1-p_2)^2}\bar{u}(p_2)\gamma_\mu u(p_1)J_\mu(p_2-p_1;p_V)\,. \quad (9)$$

However, in the presence of a Coulomb field, one must rely on the solution of the Dirac equation, rather than on the plane waves,

$$i\,\frac{\partial}{\partial t}\,\Psi(\vec{r},t) = \left[-i\,\vec{\alpha}\cdot\vec{\nabla}+\beta\,m+e\,\phi(r)\right]\,\Psi(\vec{r},t)\,. \quad (10)$$

Here $\vec{\alpha}$ and β are the standard Dirac matrixes; $\phi(r)$ is the Coulomb potential which is independent of energy in the rest frame of the nucleus; m is the lepton mass. The initial state leptonic wave function which is a solution of this equation, has the form of the sum of a plane and a spherical outgoing wave, $\Psi^+(\vec{p}_1,\vec{r})$ at $\vec{r}\to\infty$, while the final state wave function should contain a plane and an incoming wave, $\Psi^-(\vec{p}_2,\vec{r})$.

In the general case, eq. (10) can be solved only numerically, but if the effects of nuclear size can be neglected, *i.e.*

$$\phi(r) = -\frac{Ze}{r}\,, \quad (11)$$

an approximate analytical solution exists, as it was found by W.H. Furry [8],

$$\Psi^+(\vec{p}_1,\vec{r}) = \frac{C}{\sqrt{2\epsilon_1}}\,e^{i\vec{p}_1\cdot\vec{r}}\left(1-\frac{i\vec{\alpha}\cdot\vec{\nabla}}{2\epsilon_1}\right) \times F\left[iZ\alpha,1;i(p_1r-\vec{p}_1\cdot\vec{r})\right]\,u(\vec{p}_1)\,, \quad (12)$$

$$\Psi^-(\vec{p}_2,\vec{r}) = \frac{C^*}{\sqrt{2\epsilon_2}}\,e^{i\vec{p}_2\cdot\vec{r}}\left(1-\frac{i\vec{\alpha}\cdot\vec{\nabla}}{2\epsilon_2}\right) \times F\left[-iZ\alpha,1;-i(p_2r+\vec{p}_2\cdot\vec{r})\right]u(\vec{p}_2)\,, \quad (13)$$

where

$$C = e^{\pi Z\alpha/2}\,\Gamma(1 - iZ\alpha)\,,$$
$$\epsilon_{1,2} = \sqrt{\vec p_{1,2}^{\,2} + m^2}\,, \tag{14}$$

$u(\vec p_{1,2})$ are the 4-spinors of the leptons, $\Gamma(x)$ is the gamma-function, and $F(a, b; c)$ is the confluent hypergeometric function. The condition that the leptons are ultrarelativistic, $\epsilon_{1,2} \gg m$, implies that the essential orbital momenta are large, $l \gg 1$. It is well satisfied in all cases we are interested in.

For the sake of simplicity, we will work with "spinless leptons" since we are interested only in an estimate of the effects and do not expect a principal modification related to the lepton spin. Then, the lepton wave functions satisfy the Klein-Gordon equation

$$-\Delta\Psi(r) = \left\{[\epsilon - e\,\phi(r)]^2 - m^2\right\}\Psi(r) = \left[p^2 - 2\,e\,\epsilon\,\phi(r) + e^2\,\phi^2(r)\right]\Psi(r)\,. \tag{15}$$

Apparently, at high energies the term $e^2\,\phi^2(r)$ can be neglected, and for the Coulomb potential, eq. (11), one can get the exact solution

$$\Psi^+(\vec p_1, \vec r) = CF\left[iZ\alpha, 1; i(p_1 r - \vec p_1\cdot\vec r)\right] e^{i\vec p_1\cdot\vec r}\,, \tag{16}$$
$$\Psi^-(\vec p_2, \vec r) = C^* F\left[-iZ\alpha, 1; -i(p_2 r + \vec p_2\cdot\vec r)\right] e^{i\vec p_2\cdot\vec r}\,. \tag{17}$$

Correspondingly, for the lepton current, $j_\mu(x) = j_\mu(\vec r, t)$,

$$j_\mu(\vec r, t) = e^{i(\epsilon_1-\epsilon_2)t}\,j_\mu(\vec r)\,, \tag{18}$$

where

$$j_0(\vec r) = e\Psi^{-*}(\vec p_2, \vec r)\left(\epsilon_1 + \epsilon_2 + \frac{2Z\alpha}{r}\right)\Psi^+(\vec p_1, \vec r)\,, \tag{19}$$
$$\vec j(\vec r) = ie\left[\vec\nabla\Psi^{-*}(\vec p_2, \vec r)\,\Psi^+(\vec p_1, \vec r) - \Psi^{-*}(\vec p_2, \vec r)\,\vec\nabla\Psi^+(\vec p_1, \vec r)\right]. \tag{20}$$

This current is conserved

$$\frac{\partial j_0(\vec r, t)}{\partial t} = \vec\nabla\cdot\vec j(\vec r, t)\,, \tag{21}$$

or, in momentum representation

$$\nu\, j_0(Q) = \vec q\cdot\vec j(\vec q)\,, \tag{22}$$

where ν, $\vec q$ are the energy and momentum transferred to the target, $\nu^2 - \vec q^{\,2} = -Q^2$, and the Fourier transform of the current reads:

$$j_\mu(Q) = \int d^4x e^{iQx} j_\mu(x)\,. \tag{23}$$

In terms of time-independent current it reads:

$$j_0(Q) = 2\pi\delta(\epsilon_1 - \epsilon_2 - \nu)\int d^3r e^{i\vec q\cdot\vec r}\,j_0(\vec r)\,; \tag{24}$$
$$\vec j(Q) = 2\pi\delta(\epsilon_1 - \epsilon_2 - \nu)\int d^3r e^{i\vec q\cdot\vec r}\,\vec j(\vec r)\,. \tag{25}$$

Using these expressions and conservation of the hadronic current

$$\nu\, J_0(Q) = \vec q\cdot\vec J(Q)\,, \tag{26}$$

we arrive at the following form of the amplitude, eq. (4):

$$M = 2\pi\int\frac{d^3q}{-Q^2 - io}\left[\vec j(\vec q) - j_0(\vec q)\frac{\vec q}{\nu}\right]\vec J(Q)\,. \tag{27}$$

The integral in eq. (27) contains hadronic current $\vec J(Q)$ with unknown dependence on Q (Q-dependence of the leptonic current is fixed by eqs. (24),(25)). As the first simple trial we assume that it is Q-independent, $\vec J(Q) = \vec c$. Then the amplitude, eq. (27), takes the form

$$M = 2i\,(2\pi)^2\,\frac{\vec c\cdot\vec d}{\nu}\,, \tag{28}$$

where

$$\vec d = \frac{1}{2}\int d^3r\frac{e^{i\nu r}}{r}\left[i\,\vec\nabla\, j_0(\vec r) + \nu\,\vec j(\vec r)\right]. \tag{29}$$

Let us consider kinematics when the virtual photon takes a finite fraction $y = \nu/\epsilon_1$ of the initial lepton energy, and the scattering angle is small, $\theta \ll 1$. In this case, the relative contribution of the the term $Z\alpha/r$ in the current, eq. (19), estimated with the plane wave approximation, is small, $\ln(1/\theta)\,\theta^2/y \ll 1$. Thus, it is suppressed by a factor $x_{Bj}\,m_N/\nu$. Neglecting this term, we simplify the current, eq. (19), to

$$j_0(r) = (\varepsilon_1 + \varepsilon_2)\,\Psi^{-*}(\vec p_2, \vec r)\,\Psi^+(\vec p_1, \vec r)\,. \tag{30}$$

Applying eqs. (16),(17) to eqs. (20),(30), we get for the vector $\vec d$, eq. (29),

$$\vec d = \int d^3r\frac{e^{i\nu r + i\vec q\cdot\vec r}}{r}\big[(\epsilon_1\vec p_2 - \epsilon_2\vec p_1)\,F_1(\vec r)F_2(\vec r) + i\epsilon_1 F_1(\vec r)\vec\nabla F_2(\vec r) + i\epsilon_2\vec\nabla F_1(\vec r)F_2(\vec r)\big]\,, \tag{31}$$

where

$$F_1(\vec r, \vec p_1) = CF[iZ\alpha, 1; i(p_1 r - \vec p_1\cdot\vec r)]\,,$$
$$F_2(\vec r, \vec p_2) = C^* F[-iZ\alpha, 1; -i(p_2 r + \vec p_2\cdot\vec r)]\,. \tag{32}$$

Then, we make use of the following relations:

$$\vec\nabla_{\vec r} F_1(\vec r, \vec p_1) = -\frac{p_1}{r}\,\vec\nabla_{\vec p_1} F_1(\vec r, \vec p_1); \tag{33}$$
$$\vec\nabla_{\vec r} F_2(\vec r, \vec p_2) = \frac{p_2}{r}\,\vec\nabla_{\vec p_2} F_2(\vec r, \vec p_2); \tag{34}$$
$$\frac{1}{r} = \int_0^\infty d\lambda\, e^{-\lambda r}\,; \tag{35}$$

and (see in [14])

$$\int\frac{d^3r}{r}F_1(\vec r, \vec p_1)\,F_2(\vec r, \vec p_2)\,\exp(i\vec q\cdot\vec r - \lambda r + i\nu r) \equiv I(\vec q, \vec p_1, \vec p_2, \lambda) = \frac{4\pi N}{w}\left(\frac{w}{z}\right)^{iZ\alpha} F(iZ\alpha, 1 - iZ\alpha; 1; x)\,, \tag{36}$$

where $F(a,b;c;d)$ is the conventional hypergeometric function:

$$x = 1 - \frac{uv}{wz}\,;$$
$$w = \vec{q}^{\,2} + (\lambda - i\nu)^2\,;$$
$$u = (\vec{p}_2 + \vec{q})^2 - (p_2 + \nu + i\lambda)^2\,;$$
$$v = -(\vec{p}_1 - \vec{q})^2 + (p_1 + \nu + i\lambda)^2\,;$$
$$z = (p_1 + p_2 + \nu + i\lambda)^2 - (\vec{p}_1 - \vec{p}_2 - \vec{q})^2\,;$$
$$N = \left|\Gamma(1 - iZ\alpha)\right|^2 = \frac{\pi Z\alpha}{\sinh(\pi Z\alpha)}\,. \tag{37}$$

With eqs. (33),(37) we arrive at a new expression for $\vec{d}$ in eq. (31),

$$\vec{d} = (\epsilon_1 \vec{p}_2 - \epsilon_2 \vec{p}_1) \times I(\vec{q}, \vec{p}_1, \vec{p}_2, \lambda{=}0)\Big|_{\vec{q}=\vec{p}_1-\vec{p}_2} - \int\limits_0^\infty \mathrm{d}\lambda\, \vec{g}(\vec{p}_1, \vec{p}_2, \lambda)\,, \tag{38}$$

where

$$\vec{g}(\vec{p}_1, \vec{p}_2, \lambda) = i(p_2\epsilon_1 \vec{\nabla}_{\vec{p}_2} - p_1\epsilon_2 \vec{\nabla}_{\vec{p}_1})\, I(\vec{q}, \vec{p}_1, \vec{p}_2, \lambda)\Big|_{\vec{q}=\vec{p}_1-\vec{p}_2} \tag{39}$$

Further calculations are moved to Appendix A where they are performed for a photon of mass m_γ. In the final eq. (A.26) the second term in the brackets vanishes since $F(1 + iZ\alpha, 1 - iZ\alpha; 2; \tilde{x}_0)$ diverges logarithmically, but $1 - \tilde{x}_0 \propto m_\gamma^2 \to 0$. The first term at $m_\gamma \to 0$ gets

$$F(iZ\alpha, -iZ\alpha; 1; \tilde{x}_0)_{m_\gamma \to 0} \longrightarrow \frac{1}{|\Gamma(1 - iZ\alpha)|^2} = \frac{1}{N}\,. \tag{40}$$

Then eq. (A.26) leads in the limit $m_\gamma \to 0$ to

$$\vec{d} = \frac{4\pi}{Q^2}(\epsilon_1 \vec{p}_2 - \epsilon_2 \vec{p}_1) \left[\frac{Q^2}{(p_1 + p_2 + \nu)^2}\right]^{iZ\alpha}. \tag{41}$$

Thus, we have proven that the amplitude is different from the Born one only by the phase factor.

3 Coherent electroproduction of vector mesons

3.1 VDM, a point-like nucleus

Apparently, the hadronic current depends on Q^2. For instance, the VDM suggests

$$\vec{J}(Q) = \vec{J}(0)\, \frac{m_V^2}{Q^2 + m_V^2}\,, \tag{42}$$

where m_V is the vector meson mass. Including the propagator of the virtual photon $1/Q^2$ it can be represented as

$$\frac{m_V^2}{Q^2\,(Q^2 + m_V^2)} = \frac{1}{Q^2} - \frac{1}{Q^2 + m_V^2}\,. \tag{43}$$

Correspondingly, the new vector $\vec{d}$ is equal to the difference between eq. (38) and the same expression, but with replacement $\nu \Rightarrow \sqrt{\nu^2 - m_V^2}$. As a result, on top of the phase factor the amplitude acquires another complex form factor,

$$S(Q^2) = 1 - \frac{\pi Z\alpha}{\sinh(\pi Z\alpha)}\, x^{1 - iZ\alpha}\, W(\kappa)\,, \tag{44}$$

where

$$W(\kappa) = F(iZ\alpha, -iZ\alpha; 1; \kappa) - iZ\alpha(1 - \kappa)\, F(1 + iZ\alpha, 1 - iZ\alpha; 2; \kappa)\,, \tag{45}$$
$$\kappa = \frac{Q^2}{Q^2 + m_V^2}\,.$$

We calculated the ratio of the cross-sections of reaction $lA \to l'VA$ calculated with the distorted Coulomb wave functions and in the Born approximation. The results are plotted in fig. 2 as functions of $Z\alpha$ for different values of Q^2/m_V^2. Deviation from unity increases with Q^2, and is of course larger for heavier nuclei. Note that in the limit $Z\alpha \to 0$ the form factor, eq. (44), recovers the conventional VDM form, eq. (42), and the deviation from the Born approximation vanishes, as fig. 2 confirms.

If the hadronic current contains a higher power of Q^2

$$J(Q) = J(0)\, \left(\frac{m_V^2}{m_V^2 + Q^2}\right)^n\,, \tag{46}$$

it can be treated in a similar way, provided that n is integer. In this case, the previously obtained expressions should be differentiated in the parameter m_V^2.

3.2 More realistic hadronic current

We assumed above for the sake of possibility of analytical calculations that the hadron current $\vec{J}(Q)$ is independent of other variables, but Q. This means, in particular, that the amplitude of virtual coherent photoproduction $\gamma^* a \to VA$ is isotropic, *i.e.* independent of the momentum transfer $\vec{\Delta} = \vec{Q} - \vec{p}_V$. This is quite an unrealistic suggestion, and a better form of the hadronic current would be

$$\vec{J}(Q, \vec{\Delta}) = \frac{\vec{e}_V\, m_V^2}{m_V^2 + Q^2}\, \frac{1}{1 + B\,\vec{\Delta}^2/2}\,, \tag{47}$$

where $\vec{e}_V$ is the polarization vector of the vector meson, and B is the slope of the transverse momentum distribution for the reaction $\gamma^* A \to VA$.

In this case all multiple integrations in the amplitude of electroproduction of vector meson can be done analytically down to the one dimensional integral,

$$M(lA \to l'VA) = \int\limits_0^1 \mathrm{d}x\, \vec{e}_V \cdot \vec{h}(\vec{p}_1, \vec{p}_2, \vec{p}_V, x)\,, \tag{48}$$

where $\vec{h} = \vec{h}_1 - \vec{h}_2$, and

$$\vec{h}_{1,2}(\vec{p}_1, \vec{p}_2, \vec{p}_V, x) = \frac{1}{2\,\omega_{1,2}} \left\{ \left[\epsilon_2\, p_1\, \vec{\nabla}_{p_1} - \epsilon_1\, p_2\, \vec{\nabla}_{p_2} + (\epsilon_1 \vec{p}_2 - \epsilon_2 \vec{p}_1)\, \frac{\partial}{\partial\,\omega_{1,2}} \right] \times J(\vec{p}_1, \vec{p}_2, \vec{q}, \omega_{1,2}) \right\}_{\vec{q} = \vec{p}_1 - \vec{p}_2 - x\vec{p}_V} . \quad (49)$$

Here

$$\begin{aligned} x &= 1 - \frac{uv}{wz}\,; \\ w &= \vec{q}^{\,2} - \omega^2\,; \\ u &= (\vec{p}_2 + \vec{q})^2 - (p_2 + \omega)^2\,; \\ v &= (p_1 + \omega)^2 + (\vec{p}_1 - \vec{q})^2\,; \\ z &= (p_1 + p_2 + \omega)^2 - (\vec{p}_1 - \vec{p}_2 - \vec{q}2\,; \\ \omega_1 &= (1-x)\,\nu^2 - x(1-x)\,p_V^2 - x\,B/2\,; \\ \omega_2 &= (1-x)(\nu^2 - m_V^2) - x(1-x)\,p_V^2 - x\,B/2\,; \end{aligned} \quad (50)$$

and

$$\begin{aligned} J(\vec{p}_1, \vec{p}_2, \vec{q}, \omega_{1,2}) = \int \frac{\mathrm{d}^3 r}{r}\, e^{i\omega r + i\vec{q}\cdot\vec{r}} F(iZ\alpha, 1, ip_1 r - i\vec{p}_1 \cdot \vec{r}) \\ \times F(iZ\alpha, 1, ip_2 r + i\vec{p}_2 \cdot \vec{r}) = \\ \frac{4\pi N}{w} \left(\frac{w}{z}\right)^{iZ\alpha} F(iZ\alpha, 1 - iZ\alpha; 1; x). \end{aligned} \quad (51)$$

We performed numerical calculations for forward production ($\vec{\Delta} = 0$) of transversely polarized vector mesons. The ratio of the calculated and Born cross-sections $\sigma/\sigma_{\mathrm{Born}} = |M|^2/|M_{\mathrm{Born}}|^2$ is depicted in fig. 3 for few values of ϵ_1 at fixed $y = (\epsilon_1 - \epsilon_2)/\epsilon_1 = 0.6$ and $Q^2 = m_\rho^2$.

The magnitude of the effect is probably overestimated since the nucleus size is taken into account only in the production amplitude $\gamma^* A \to VA$, while the wave functions of the incident and scattered leptons still correspond to the potential of a point-like center (see next sect. 3.3). Nevertheless, these calculations reveal a new effect, the Coulomb corrections can be either positive, like in the previous case, or negative (compare with the results of calculations in [15]).

We have also studied how these Coulomb corrections vary as functions of y and Q^2. While the magnitude of deviation from the Born cross-section is nearly independent of y, it substantially increases with Q^2, keeping about the same shape of the dependence on $Z\alpha$.

3.3 Effects of nuclear size on the lepton wave functions

One should expect the Coulomb effects to be diminished when the size of the nucleus is taken into account. Indeed, the V-meson interaction radius is short and the beam lepton has to have about the same impact parameter as the virtual photon or V-meson (the difference is $\sim 1/Q$). Propagating through the nucleus, the lepton experiences a weaker electric field compared to the case of a point-like Coulomb center. Correspondingly, the Coulomb wave functions of the leptons must be corrected for the finite size of the nucleus.

The general case of a massless lepton in an isotropic potential $V(r)$ has been solved and the radial wave functions, correct in all orders in $Z\alpha$, have been found in [16] within the quasiclassical WKB approximation. They have been used later in [17–19] to sum up the partial waves and build a formal expression for the lepton wave functions

$$\Psi^{\pm}(\vec{p}, \vec{r}) = e^{\pm i\delta_{1/2}\,\eta(r)}\, e^{\pm ib(\vec{J}^2 - 3/4)}\, e^{i\vec{p}\cdot\vec{r}\,\eta(r)}\, u(p)\,, \quad (52)$$

where $\vec{J} = \vec{L} + \vec{\sigma}/2$ is the operator of the total angular momentum

$$\begin{aligned} b &= -\frac{1}{2\,p^2} \int_0^\infty \mathrm{d}r \frac{1}{r} \frac{\mathrm{d}V(r)}{\mathrm{d}r}\,, \\ \delta_{1/2} &= -Z\alpha\, \ln(p\,R_A) - \int_0^{R_A} \mathrm{d}r\, V(r) + b\,, \\ \eta(r) &= \frac{1}{\rho\, r} \int_0^r \mathrm{d}r' \left[p - V(r) \right], \end{aligned} \quad (53)$$

and $u(p)$ is the 4-spinor of the lepton. Equation (52) is a generalization of the Furry's wave functions, eq. (13).

Apparently, such an expression with the operator $\vec{J}$ in the exponent is not easy to use for practical applications. This is why only the first two terms of the order of $(Z\alpha)^0$ and $(Z\alpha)^1$ in the expansion of the exponential in (52) have been considered in [17–19]. We skip here those lengthy expressions, but apply the procedure developed in [17,18] to our case. We assume a homogeneous charge distribution inside a sphere of nuclear radius R_A and use the approximation $\epsilon_{1,2} R_A \gg 1$ and $\theta_{12} \ll 1$, where θ_{12} is the lepton scattering angle. Then the ratio of the amplitude modified by the Coulomb field to the Born one takes the form

$$\frac{M(lA \to l'VA)}{M_{\mathrm{Born}}(lA \to l'VA)} = 1 + i\, \frac{3B\, Z\alpha}{R_A^2} = 1 + i\, \frac{3\, Z\alpha}{5}\,. \quad (54)$$

Here B is the slope parameter of the differential cross-section introduced in (47). It is related to the mean charge nuclear radius squared:

$$B = \frac{1}{3}\, \langle r_{\mathrm{ch}}^2 \rangle_A = \frac{1}{5}\, R_A^2\,. \quad (55)$$

Thus, deviation from the Born cross-section,

$$\frac{\sigma(lA \to l'VA)}{\sigma_{\mathrm{Born}}(lA \to l'VA)} = 1 + \frac{9}{25}\, (Z\alpha)^2\,. \quad (56)$$

This is a sizeable correction for heavy nuclei, for example, the modified cross-section on lead is 13% higher than the Born one.

Surprisingly, the correction, eq. (56), does not expose any dependence on reaction kinematics, contrary to the results of the previous sections. It is probably a consequence of the higher-order terms in $Z\alpha$ missed in this calculations. How to elaborate on those terms is still a challenge which we leave for further studies. The purpose of the present estimates is to see whether the finiteness of nuclear size can substantially diminish the effect of long-range Coulomb forces. Apparently not, the deviation is still sizeable. One should calculate at least the next term $O(Z^2\alpha^2)$ to make it sure that the correction, eq. (54), is not just a phase. We believe that such a possibility is very improbable, it happened so far only in the case of elastic scattering, and in the special artificial case when the hadronic current is a constant (sect. 2). In both cases, the Furry wave functions for a point-like Coulomb center are used. The example of sect. 3.1 demonstrates that even the simplest Q^2-dependence of the hadronic current leads to an amplitude which consists of few terms having different phases, and the cross-section changes.

4 Eikonal approximation

A different approach to the problem of distortion of the lepton wave functions in the Coulomb field of an extended nucleus is the eikonal approximation of Bjorken, Kogut and Soper (BKS) [20]. It is not clear how precise this approximation is, the best way to figure it out is to compare its results with the known exact solution in the case when it is available. The exact cross-section has been calculated by Davies, Bethe and Maximon (DBM) [21] for photoproduction of e^+e^- pairs and Bethe and Maximon for bremsstrahlung [10]. Using the BKS approach, we calculate in Appendix B the cross-section for this reaction and arrive at exactly the same Coulomb correction to the Born cross-section, as it has been found in [21,10]. This success can serve as an argument that the eikonal approximation is rather accurate.

In the eikonal approximation the wave functions of spinless leptons read:

$$\Psi^+(\vec{p}_1,\vec{r}) = \exp\Big[i\,\vec{p}_1\cdot\vec{r} - i\,\chi_1(\vec{r})\Big]\,;$$
$$\Psi^-(\vec{p}_2,\vec{r}) = \exp\Big[i\,\vec{p}_2\cdot\vec{r} + i\,\chi_2(\vec{r})\Big]\,. \qquad (57)$$

Here the phase shifts are

$$\chi_1(\vec{r}) = \int\limits_{-\infty}^{z} \mathrm{d}z'\,V(\vec{b},z')\,,$$
$$\chi_2(\vec{r}) = \int\limits_{z}^{\infty} \mathrm{d}z'\,V(\vec{b},z')\,. \qquad (58)$$

At high energies, $\epsilon_{1,2} \gg \sqrt{Q^2}$, the vectors $\vec{p}_1$, $\vec{p}_2$ and $\vec{p}_1 - \vec{p}_2$ are nearly parallel. We chose the axis z along $\vec{p}_1$, correspondingly the vector $\vec{r} = (\vec{b},z)$ has projection $\vec{b}$ to the normal plane.

For the long-range Coulomb field, $V(r)_{r\to\infty} = \pm Z\alpha/r$, the integrals in (58) are strictly speaking divergent. To fix the problem we introduce an infra-red cut-off

$$V(r)\Big|_{r\to\infty} = \pm\,\frac{Z\alpha}{r}\,e^{-\lambda r}\,\left(1 - e^{-\mu r}\right)\,, \qquad (59)$$

where the last factor corresponds to the pole form of the nuclear form factor with

$$\mu^{-2} = \frac{\langle r^2_{\mathrm{ch}}\rangle_A}{6}\,. \qquad (60)$$

One may interpret λ in (59) as an effective photon mass, or (better justified) as the inverse screening radius of the nuclear Coulomb field by the atomic electrons. As soon as $1/\lambda \gg R_A$, any variation of the value of λ may lead only to r-independent additive corrections to the phase shifts $\chi_{1,2}(\vec{r})$ which does not affect the value of the cross-section we are interested in.

Utilizing the same realistic form of the hadronic current for electroproduction of vector mesons as in sect. 3.2, we get the following expression for the amplitude:

$$M(lA \to l'VA) = \int\limits_0^1 \mathrm{d}x\,\vec{e}_V\cdot\vec{f}(\vec{p}_1,\vec{p}_2,\vec{p}_V,x)\,, \qquad (61)$$

where

$$\vec{f} = \vec{f}_1 - \vec{f}_2\,,$$

and

$$\vec{f}_{1,2}(\vec{p}_1,\vec{p}_2,\vec{p}_V,x) = \frac{1}{2\omega_{1,2}}\,\frac{\partial}{\partial\omega_{1.2}}$$
$$\times\int\frac{\mathrm{d}^3r}{r}\,\Big\{\Big[\epsilon_1\vec{p}_2 - \epsilon_2\vec{p}_1\Big] - \Big[\epsilon_1\vec{\nabla}\,\chi_2(\vec{r}) + \epsilon_2\vec{\nabla}\,\chi_1(\vec{r})\Big]\Big\}$$
$$\times\exp\Big[i\vec{\kappa}\vec{r} - i\chi_1(\vec{r}) - i\chi_2(\vec{r}) + i\,\omega_{1,2}\,r\Big]\,, \qquad (62)$$

where $\vec{\kappa} = \vec{p}_1 - \vec{p}_2 - x\,\vec{p}_V$.

Note that the x-dependence of this expression comes via $\omega_{1,2}$ defined in (50). The derivatives $\vec{\nabla}\chi_{1,2}(r)$ in (62) are the momenta transferred by the Coulomb field to the initial and final leptons. In the case of an extended nucleus they are of the order of $Z\alpha/R_A$.

Using the relation

$$\Big|\epsilon_1\vec{p}_2 - \epsilon_2\vec{p}_1\Big| = \sqrt{\epsilon_1\epsilon_2\,Q^2}\,, \qquad (63)$$

we see that for $Q^2 \gg R_A^{-2}$, the second term in curly brackets in (62) is much smaller than the first one and can be neglected. Then eq. (58) takes the form

$$\chi_1(\vec{r}) + \chi_2(\vec{r}) = \int\limits_{-\infty}^{\infty}\mathrm{d}z\,V(\vec{b},z) = \chi(\vec{b}) =$$
$$\pm 2Z\alpha\Big\{K_0(\lambda b) - K_0[(\mu+\lambda)b]\Big\}\,, \qquad (64)$$

where plus and minus correspond to different signs of the potential, eq. (59). Since the sum $\chi_1(\vec{r}) + \chi_2(\vec{r})$ is independent of z, one can explicitly perform integration over

z in eq. (62), using the following relation:

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}z}{r} \exp(i\,\kappa_{\mathrm{L}} z + i\omega r) = 2\,K_0\Big(b\,\sqrt{\kappa_{\mathrm{L}}^2 - \omega^2}\Big)\,, \qquad (65)$$

where $K_0(x)$ is the modified Bessel function, $\omega = \omega_{1,2}$, and $\kappa_{\mathrm{L,T}}$ are the longitudinal and transverse components of $\vec{\kappa}$.

Thus, the problem of the calculation of the amplitude (61) is reduced to integrals of the form

$$\int_0^1 \mathrm{d}x \int_0^{\infty} \mathrm{d}b\, b\, K_0\Big(b\,\sqrt{\kappa_{\mathrm{L}}^2 - \omega^2}\Big)\, e^{i\chi(b)}\, J_0(\kappa_{\mathrm{T}} b)\,, \qquad (66)$$

which we calculate numerically. Our results for ratio of the calculated and Born cross-sections are depicted in fig. 4 as functions of $Z\alpha$.

It turns out that the cross-section is smaller than the Born one, and the difference is Q^2-independent at $Q^2 > 1$ GeV^2. However, the range of Q^2 correlates with the photon energy ν to satisfy the condition of coherence $q_L = (Q^2 + m_V^2)/2\nu \lesssim 1/R_A$.

5 Conclusions and outlook

The Born approximation employed in all analyses of experimental data has no justification for heavy nuclei with $Z\alpha \sim 1$. Therefore, the accuracy of experimental results for nuclear effects in DIS may be essentially affected by this theoretical uncertainty.

Radiative corrections are known to provide a substantial contribution to the DIS cross-section. Usually they can be calculated rather accurately [22], except one related to the modification of the lepton wave function by long-range Coulomb forces. This correction violating the Born approximation is usually ignored, and for a good reason: it is a very difficult task to calculate it. Nevertheless this problem should be challenged rather than "swept under the carpet".

A certain progress is made in the present paper towards the calculation of the DIS cross-section using the lepton wave functions modified by the long-range Coulomb forces. We start with the simplest assumption that the hadronic current is independent of Q^2 and prove that the DIS amplitude acquires only a phase. In this case the Coulomb corrections do not modify the Born cross-section.

This conclusions, however, changes after a Q^2-dependence is introduced into the hadronic current. We demonstrate that on the example of coherent leptoproduction of vector mesons off a point-like nucleus which we treat within VDM. Of course, VDM should not be used at high Q^2 where color transparency is important. However, we just want to estimate the scale of the correction, while precise calculations ready to use in an analysis of data are still a challenge.

Further steps beyond the approximation of a point-like target lead to a conclusion that, in the case of an extended nucleus, the Coulomb corrections are still important and can affect the existing experimental results for nuclear effects in DIS which are based on the Born approximation. To do more precise calculations one needs to know the lepton wave functions in the Coulomb field of an extended nucleus which are currently available only in the first order in $Z\alpha$.

We also use the eikonal approximation which is better designed for the case of extended nuclei. First, we demonstrate (Appendix B) that this approximation well reproduces the known exact results when they are available. Then we apply this method to the process of coherent electroproduction of vector mesons and again arrive at a sizeable correction to the Born cross-section.

We conclude that the long-range Coulomb forces may significantly modify the DIS cross-section on heavy nuclei compared to the widely used Born approximation. Further progress in this direction should help to make experimental results for nuclear effects in DIS more reliable.

Although originally our work has been motivated by the unusual nuclear effects in DIS, observed in the HERMES experiment [12] (see Introduction), we did not find anything special about the energy and x range of this data. Whatever deviations from the Born approximation happen due to the long-range Coulomb forces, they should have similar magnitude either at the energy of HERMES, or NMC experiments.

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Appendix A. Calculation of the vector $\vec{d}$

We present here the details of calculations of the vector $\vec{d}$ in eq. (38). Since we are going to use the results also in sect. 3.1, where the amplitude contains two terms in eq. (43), we will treat the photon as a massive particle with mass m_γ which is either zero (the first term in (43)), or equal to m_V (the first term in (43)). Correspondingly, in all equations of sect. 2 one should replace $\nu \Rightarrow \tilde{\nu} = \sqrt{\nu^2 - m_\gamma^2}$.

The expression for the vector $g(\vec{p}_1, \vec{p}_2, \lambda)$ in eq. (39) can be represented as

$$\vec{g}(\vec{p}_1, \vec{p}_2, \lambda) = \Big[\Big(p_2\epsilon_1\,\vec{\nabla}\cdot\vec{p}_2 - p_1\epsilon_2\,\vec{\nabla}\cdot\vec{p}_1\Big)\, I(\vec{q}, \vec{p}_1, \vec{p}_2, \lambda)\Big]_{\vec{q}=\vec{p}_1-\vec{p}_2} + \frac{4\pi i N}{Q^2 + m_\gamma^2 + \lambda^2 - 2i\tilde{\nu}} \left[\frac{Q^2 + m_\gamma^2 + \lambda^2 2 - 2i\tilde{\nu}}{(p_1 + p_2 + \tilde{\nu} + i\lambda)^2}\right]^{iZ\alpha}$$

$$\times\left\{-\frac{2iZ\alpha(\epsilon_1\vec{p}_2-\epsilon_2\vec{p}_1)}{p_1+p_2+\tilde{\nu}+i\lambda}\,\Phi(\tilde{x})-2\left(\frac{\epsilon_1}{D_1}-\frac{\epsilon_2}{D_2}\right)\right.$$
$$\left.\times(p_1\vec{p}_2-p_2\vec{p}_1)(1-\tilde{x})\,\Phi'(\tilde{x})\right\}. \tag{A.1}$$

The following notations are used here:

$$\Phi(\tilde{x})=F(iZ\alpha,1-iZ\alpha;1;\tilde{x})\,; \tag{A.2}$$
$$\Phi'(\tilde{x})=\frac{\mathrm{d}}{\mathrm{d}\tilde{x}}\,\Phi(\tilde{x})\,; \tag{A.3}$$
$$\tilde{x}=x_{\vec{q}=\vec{p}_1-\vec{p}_2}=\frac{Q^2-\Delta^2}{Q^2+m_\gamma^2+\lambda^2-2i\tilde{\nu}\lambda}\,; \tag{A.4}$$
$$Q^2=\vec{q}^{\,2}-\nu^2\,;\qquad \Delta^2=(p_1-p_2)^2-\nu^2\,; \tag{A.5}$$
$$D_1=u_{\vec{q}=\vec{p}_1-\vec{p}_2}=p_1^2-(p_2+\tilde{\nu}+i\lambda)^2\,;\qquad D_2=v_{\vec{q}=\vec{p}_1-\vec{p}_2}=(p_1+\tilde{\nu}+i\lambda)^2-p_2^2\,. \tag{A.6}$$

Disregarding the difference between $p_{1,2}$ and $\epsilon_{1,2}$ (equivalent to dropping off the corrections of the order of $m_\gamma^2/p_{1,2}^2$), we can write

$$\frac{2\epsilon_1}{D_1}-\frac{2\epsilon_2}{D_2}=\frac{2(\tilde{\nu}+i\lambda)}{(p_1-p_2)^2+(\lambda-i\tilde{\nu})^2}-\frac{2}{p_1+p_2+\nu+i\lambda}\,. \tag{A.7}$$

Making use of this relation, we get

$$\vec{g}(\lambda)=\frac{4\pi N\,(\epsilon_1\vec{p}_1-\epsilon_2\vec{p}_1)}{Q^2+m_\gamma^2+\lambda^2-2i\tilde{\nu}\lambda}\left[\frac{Q^2+m_\gamma^2+\lambda^2-2i\tilde{\nu}\lambda}{(p_1+p_2+\tilde{\nu}+i\lambda)2}\right]^{iZ\alpha}$$
$$\times\left[\frac{2(\lambda-i\tilde{\nu})}{(p_1-p_2)^2+(\lambda-i\tilde{\nu})^2}(1-\tilde{x})\Phi'(\tilde{x})\right.$$
$$\left.+\frac{2Z\alpha\,\Phi(\tilde{x})+2i(1-\tilde{x})\,\Phi'(\tilde{x})}{p_1+p_2+\tilde{\nu}+i\lambda}\right]. \tag{A.8}$$

Notice that

$$\frac{2(\lambda-i\tilde{\nu})(1-\tilde{x})}{(\vec{p}_1-\vec{p}_2)^2+(\lambda+i\tilde{\nu})^2}=$$
$$\frac{2(\lambda-i\nu)}{Q^2+m_\gamma^2+\lambda^2-2i\nu\lambda}=-\frac{1}{\tilde{x}}\frac{\partial\tilde{x}}{\partial\lambda}\,, \tag{A.9}$$

and

$$2Z\alpha\,\Phi(\tilde{x})+2i(1-\tilde{x})\,\Phi'(\tilde{x})=i(Z\alpha)^2\,\tilde{\Phi}(\tilde{x})\,, \tag{A.10}$$

where

$$\tilde{\Phi}(\tilde{x})=F(1+iZ\alpha,1-iZ\alpha;2;\tilde{x})\,. \tag{A.11}$$

Therefore, the vector $\vec{g}(\lambda)$ gets the form

$$\vec{g}(\lambda)=\frac{4\pi N(\epsilon_1\vec{p}_2-\epsilon_2\vec{p}_1)}{Q^2}\left[\frac{Q^2+m_\gamma^2+\lambda^2-2i\tilde{\nu}\lambda}{(p_1+p_2+\tilde{\nu}+i\lambda)^2}\right]^{iZ\alpha}$$
$$\times\left[-\frac{\partial\Phi}{\partial\lambda}+\frac{i(Z\alpha)^2\,\tilde{x}\,\tilde{\Phi}(\tilde{x})}{p_1+p_2+i\lambda+\tilde{\nu}}\right]. \tag{A.12}$$

Here, the second term in the brackets of the last factor is small, $\sim\theta^2\ln(1/\theta)$, relative to the first term (see below), therefore, it can be neglected for the small scattering angles $\theta\ll 1$. In this case, the problem under consideration is reduced to the integral

$$\int_0^\infty \mathrm{d}\lambda\,\vec{g}(\lambda)=\frac{4\pi N}{Q^2}\,(\epsilon_1\vec{p}_2-\epsilon_2\vec{p}_1)\,L \tag{A.13}$$

where

$$L=\int_0^\infty \mathrm{d}\lambda\,t^{iZ\alpha}\,\frac{\partial\Phi}{\partial\lambda}\,, \tag{A.14}$$

and

$$t=\frac{Q^2+m_\gamma^2+\lambda^2-2i\nu\lambda}{p_1+p_2+\tilde{\nu}+i\lambda}\,. \tag{A.15}$$

Integrating in eq. (A.14) by parts, we get

$$L=t^{iZ\alpha}\,\Phi(\tilde{x})\Big|_{\lambda=0}^{\lambda=\infty}-\int_0^\infty \mathrm{d}\lambda\,\frac{\partial\,t^{iZ\alpha}}{\partial\,\lambda}\,\Phi(\tilde{x})\,. \tag{A.16}$$

Since $\tilde{x}=0$ at $\lambda\to\infty$ and $\Phi(0)=1$, we have

$$t^{iZ\alpha}\,\Phi(\tilde{x})\Big|_{\lambda=0}^{\lambda=\infty}=(-1)^{iZ\alpha}-t_0^{iZ\alpha}\,\Phi(\tilde{x}_0)\,, \tag{A.17}$$

where

$$t_0=t(\lambda=0)=\frac{Q^2+m_\gamma^2}{(p_1+p_2+\tilde{\nu})^2}\,;$$
$$\tilde{x}_0=\tilde{x}(\lambda=0)=\frac{Q^2}{Q^2+m_\gamma^2}\,. \tag{A.18}$$

To calculate the rest integrals we notice that the value of $\tilde{x}$ is tiny, except for the region of $\lambda<(Q^2+m_\gamma^2)/(2\tilde{\nu})\ll 1$. Then, we split the integral in eq. (A.16) into two parts,

$$\int_0^\infty \mathrm{d}\lambda=\int_0^{\lambda_1}\mathrm{d}\lambda+\int_{\lambda_1}^\infty \mathrm{d}\lambda\,, \tag{A.19}$$

where we chose $(Q^2+m_\gamma^2)/(2\tilde{\nu})\ll\lambda_1\ll\nu$, so that $|\tilde{x}|\ll 1$ in the second term in (A.19). Therefore, we can fix $\Phi(\tilde{x})=1$ in the second integral in (A.19) and get

$$\int_{\lambda_1}^\infty \mathrm{d}\lambda\,\frac{\partial\,t^{iZ\alpha}}{\partial\,\lambda}\,\Phi(\tilde{x})\approx$$
$$t^{iZ\alpha}\Big|_{\lambda_1}^{\infty}+O\left(\frac{\lambda_1}{\nu}\right)=(-1)^{iZ\alpha}-t_1^{iZ\alpha}+O\left(\frac{\lambda_1}{\nu}\right), \tag{A.20}$$

where $t_1=t(\lambda_1)$.

In the first integral in (A.19) we can make use of the smallness of λ in the expression (A.15) and rewrite it as

$$t=\frac{Q^2+m_\gamma^2-2i\lambda\tilde{\nu}}{(p_1+p_2+\tilde{\nu})^2}=\frac{\tilde{x}_0\,t_0}{\tilde{x}}\,\frac{1}{x}\,. \tag{A.21}$$

Then, the first integral in (A.19) takes the form

$$\int_0^{\lambda_1} \mathrm{d}\lambda \frac{\partial t^{iZ\alpha}}{\partial \lambda} \Phi(\tilde{x}) =$$

$$-iZ\alpha(\tilde{x}_0 t_0)^{iZ\alpha} \int_{\tilde{x}_0}^{\tilde{x}_1} \mathrm{d}\tilde{x}\tilde{x}^{-1-iZ\alpha}\Phi(\tilde{x}) =$$

$$(\tilde{x}_0 t_0)^{iZ\alpha} \left[\tilde{x}^{-iZ\alpha} F(iZ\alpha, iZ\alpha; 1; \tilde{x})\right]\Big|_{\tilde{x}_0}^{\tilde{x}_1} =$$

$$t_1^{iZ\alpha} - t_0^{iZ\alpha} F(iZ\alpha, iZ\alpha; 1; \tilde{x}_0) + O\left(\frac{\lambda_1}{\nu}\right). \qquad \text{(A.22)}$$

Here, we used the definition of $\Phi(\tilde{x})$ from eq. (A.2).

Now, taking into account that $|\tilde{x}(\lambda = \lambda_1)| \ll 1$, we eventually arrive at the following expression for the integral, eq. (A.14):

$$L = t_0^{iZ\alpha} \left[F(iZ\alpha, iZ\alpha; 1; \tilde{x}_0) - F(iZ\alpha, 1 - iZ\alpha; 1; \tilde{x}_0)\right]. \qquad \text{(A.23)}$$

Taking into account the relation

$$F(iZ\alpha, iZ\alpha; 1; \tilde{x}_0) = F(iZ\alpha, -iZ\alpha; 1; \tilde{x}_0) + iZ\alpha\, \tilde{x}_0\, F(1 + iZ\alpha, 1 - iZ\alpha; 2; \tilde{x}_0), \qquad \text{(A.24)}$$

we can integrate $\vec{g}(\lambda)$ as

$$\int_0^\infty \mathrm{d}\lambda\, g(\lambda) = \frac{4i\pi(\epsilon_1\vec{p}_2 - \epsilon_2\vec{p}_1)}{Q^2 + m_\gamma^2} \left[\frac{Q^2 + m_\gamma^2}{(p_1 + p_2 + \tilde{\nu})^2}\right]^{iZ\alpha} \times iZ\alpha\, F(1 + iZ\alpha, 1 - iZ\alpha; 2; \tilde{x}_0). \qquad \text{(A.25)}$$

Substituting this expression into eq. (38), we eventually arrive at the final expression for $\vec{d}$:

$$\vec{d} = \frac{4\pi\, N}{Q^2 + m_\gamma^2} (\epsilon_1\vec{p}_2 - \epsilon_2\vec{p}_1) \left[\frac{Q^2 + m_\gamma^2}{(p_1 + p_2 + \tilde{\nu})^2}\right]^{iZ\alpha} \times \left[F(iZ\alpha, -iZ\alpha; 1; \tilde{x}_0) - iZ\alpha(1 - \tilde{x}_0) \times F(1 + iZ\alpha, 1 - iZ\alpha; 2; \tilde{x}_0)\right]. \qquad \text{(A.26)}$$

Appendix B. Eikonal approximation vs. the exact calculations

Here we calculate the total cross-section of photoproduction of electron-positron pairs off atoms at high energies relying upon the eikonal approximation of ref. [20] and compare with the exact results of [21].

According to [20] the photoproduction cross-section has the factorized form

$$\sigma(\gamma Z \to e^+e^- Z) = \int_0^1 \mathrm{d}u \int \mathrm{d}^2\rho\sigma(\rho, u) \left|\Psi(\vec{\rho}, u)\right|^2, \qquad \text{(B.1)}$$

where $\sigma(\rho, u)$ is the cross-section of interaction of the e^+e^- dipole with the atom which depends on the transverse separation $\vec{\rho}$ and the fraction $u = (E + p_L)/2\omega$ of the photon light cone momentum carried by the electron. $\Psi(\vec{\rho}, u)$ is the e^+e^- wave function of the photon

$$\left|\Psi(\vec{\rho}, u)\right|^2 = \frac{\alpha\, m^2}{2\pi^2} \left\{K_0^2(m\rho) + \left[u^2 + (1-u)^2\right] K_1^2(m\rho)\right\}, \qquad \text{(B.2)}$$

where m is the electron mass, α is the fine structure constant.

If the atom remains intact, then according to [20],

$$\sigma(\rho, u) = 2 \int \mathrm{d}^2b \left\{1 - \exp\left[i\chi(\vec{b} - u\vec{\rho}) - i\chi(\vec{b} + \vec{\rho} - u\vec{\rho})\right]\right\}, \qquad \text{(B.3)}$$

where $\chi(\vec{b})$ is defined in (64).

Replacing in (B.3) $\vec{b} + (1/2 - u)\vec{\rho} \Rightarrow \vec{b}$ we arrive at

$$\sigma(\rho, u) = 2 \int \mathrm{d}^2b \left\{1 - \exp\left[i\chi(\vec{b}_+) - i\chi(\vec{b}_-)\right]\right\}, \qquad \text{(B.4)}$$

where $\vec{b}_\pm = \vec{b} \pm \vec{\rho}/2$. Thus, we conclude that the dipole cross-section, eq. (B.4), depends only on the transverse separation ρ.

The phase shifts $\chi(\vec{b}_\pm)$ can be expressed in terms of the transverse density of electrons in the atom $n(s)$,

$$\chi(\vec{b}_\pm) = 2Z\alpha \int_{b_\pm}^\infty \mathrm{d}s\, s\, n(s) \ln\left(\frac{s}{b_\pm}\right), \qquad \text{(B.5)}$$

where $b_\pm = |\vec{b}_\pm|$ and the density is normalized as

$$\int_0^\infty \mathrm{d}s\, s\, n(s) = 1. \qquad \text{(B.6)}$$

The atomic size $R_Z \sim 1/(m\alpha Z^{1/3})$ is much larger than the transverse e^+e^- separation $\rho \sim 1/m$, therefore, one can split the integral in (B.4) into two parts,

$$\sigma(\rho) = 2 \int_0^\infty \mathrm{d}b\, b \int_0^{2\pi} \mathrm{d}\phi \left[1 - \exp(i\chi_+ - i\chi_-)\right] = \sigma_1(\rho) + \sigma_2(\rho), \qquad \text{(B.7)}$$

where ϕ is the azimuthal angle between $\vec{b}$ and $\vec{\rho}$; $\chi_\pm = \chi(\vec{b}_\pm)$, and

$$\sigma_1(\rho) = 2 \int_0^{b_0} \mathrm{d}b\, b \int_0^{2\pi} d\phi \left[1 - \exp(i\chi_+ - i\chi_-)\right] \qquad \text{(B.8)}$$

$$\sigma_2(\rho) = 2 \int_{b_0}^\infty \mathrm{d}b\, b \int_0^{2\pi} \mathrm{d}\phi \left[1 - \exp(i\chi_+ - i\chi_-)\right]. \qquad \text{(B.9)}$$

We chose the value of b_0 satisfying the condition

$$\frac{1}{m} \ll b_0 \ll R_Z. \qquad \text{(B.10)}$$

Starting with eq. (B.9) for $\sigma_2(\rho)$ we note that the mean value of the e^+e^- separation is much smaller than the impact parameter

$$\rho \sim \frac{1}{m} \ll b_0 \leq b\,.$$

Therefore,

$$\chi_+ - \chi_- = \chi\left(\vec{b}+\frac{\vec{\rho}}{2}\right) - \chi\left(\vec{b}-\frac{\vec{\rho}}{2}\right) = \vec{\rho}\cdot\vec{\nabla}_b\,\chi(\vec{b}) + O\left(\frac{\rho^3}{b^3}\right) \approx \rho\,\cos\phi\,\frac{\mathrm{d}\chi(b)}{\mathrm{d}\,b} = 2Z\alpha\cos\phi\,\frac{\rho}{b}\int\limits_b^\infty \mathrm{d}s\,s\,n(s)\,. \quad \text{(B.11)}$$

Correspondingly,

$$\sigma_2(\rho) = 4\pi\int\limits_{b_0}^\infty \mathrm{d}b\,b\left[1-J_0\left(\rho\frac{\mathrm{d}\chi}{\mathrm{d}\,b}\right)\right] = 4\pi(Z\alpha)^2\rho^2 \times\int\limits_{b_0}^\infty\frac{\mathrm{d}\,b}{b}\left[\int\limits_b^\infty \mathrm{d}s\,s\,n(s)\right]^2 + O\left(\frac{\rho^4}{R_Z^4}\right). \quad \text{(B.12)}$$

Integrating this expression by parts, we get

$$\int\limits_{b_0}^\infty\frac{\mathrm{d}\,b}{b}\left[\int\limits_b^\infty \mathrm{d}s\,s\,n(s)\right]^2 = \ln\left(\frac{1}{b_0}\right)\left[\int\limits_{b_0}^\infty \mathrm{d}s\,s\,n(s)\right]^2 -2\int\limits_{b_0}^\infty \mathrm{d}b\,b\,\ln\left(\frac{1}{b}\right)n(b)\int\limits_b^\infty \mathrm{d}s\,s\,n(s)\,. \quad \text{(B.13)}$$

The bottom limits b_0 of the integrals in the right-hand side (r.h.s.) of (B.13) can be replaced by zero, $b_0 \Rightarrow 0$, with an accuracy of the order of $b_0^2/R_Z^2 \ll 1$. Then, taking into account the normalization condition, eq. (B.6), we get

$$\sigma_2(\rho) = 4\pi(Z\alpha)^2\,\rho^2\left[\ln\left(\frac{a}{b_0}\right) + O\left(\frac{b_0^2}{R_Z^2}\right)\right], \quad \text{(B.14)}$$

where

$$\ln a = -2\int\limits_0^\infty \mathrm{d}b\,b\,\ln\left(\frac{1}{b}\right)n(b)\int\limits_b^\infty \mathrm{d}s\,s\,n(s)\,. \quad \text{(B.15)}$$

Apparently, the value of $a \sim R_Z$ depends only on the details of the atomic structure contained in the electron density distribution $n(s)$.

Now we turn to the first term $\sigma_1(\rho)$ in (B.7). Since $b \leq b_0 \ll R_Z$, also $b_\pm = |\vec{b}\pm\vec{\rho}/2| \leq b_0 \ll R_Z$. Making use of these relations, we get

$$\chi_\pm = 2Z\alpha\int\limits_{b_\pm}^\infty \mathrm{d}s\,s\,n(s)\ln\left(\frac{s}{b_\pm}\right) = 2Z\alpha\int\limits_0^\infty \mathrm{d}s\,s\,n(s)\ln\left(\frac{s}{b_\pm}\right) + O\left(\frac{b_\pm^2}{R_Z^2}\right). \quad \text{(B.16)}$$

Consequently,

$$\chi_+-\chi_- = 2Z\alpha\ln\left(\frac{b_-}{b_+}\right)\int\limits_0^\infty \mathrm{d}s\,s\,n(s) + O\left(\frac{b_\pm^2}{R_Z^2}\right) \approx Z\alpha\ln\left[\frac{(\vec{b}-\vec{\rho}/2)^2}{\vec{b}+\vec{\rho}/2)^2}\right] = Z\alpha\ln\left(\frac{b^2-b\rho\cos\phi+\rho^2/4}{b^2+b\rho\cos\phi+\rho^2/4}\right). \quad \text{(B.17)}$$

Substituting this expression in (B.9), we get $\sigma_1(\rho)$ in the following form:

$$\sigma_1(\rho) = 2\int\limits_0^{b_0}\mathrm{d}b\,b\int\limits_0^{2\pi}\mathrm{d}\phi\left[1-\left(\frac{b^2-b\rho\cos\phi+\rho^2/4}{b^2+b\rho\cos\phi+\rho^2/4}\right)^{iZ\alpha}\right]. \quad \text{(B.18)}$$

To proceed to the further modifications of this expression, we employ the relation

$$1 - X^Y = -XY\,F(1-Y,1;2;1-X)\,, \quad \text{(B.19)}$$

with

$$X = \frac{b^2-b\rho\cos\phi+\rho^2/4}{b^2+b\rho\cos\phi+\rho^2/4}\,,\qquad Y = iZ\alpha\,. \quad \text{(B.20)}$$

We also make use of the following representation for the hypergeometric function:

$$F(1-Y,1;2;1-X) = \frac{1}{\Gamma(1-Y)\Gamma(1+Y)}\int\limits_0^1\mathrm{d}t\frac{t^{-Y}(1-t)^Y}{1-t(1-X)}\,. \quad \text{(B.21)}$$

Applying these relations to (B.18), we get

$$\sigma_1(\rho) = -\frac{2}{\Gamma(1-iZ\alpha)\Gamma(1+iZ\alpha)} \times\int\limits_0^1\mathrm{d}t\,t^{-iZ\alpha}(1-t)^{iZ\alpha}L(b_0,\rho,t)\,, \quad \text{(B.22)}$$

where

$$L(b_0,\rho,t) = \int\limits_0^{b_0}\mathrm{d}b\,b\int\limits_0^{2\pi}\mathrm{d}\phi\frac{2\,b\,\rho\,\cos\phi}{b^2+(1-2t)b\rho\cos\phi+\rho^2/4} = 2\pi\rho^2(1-2t)\left\{\frac{b_0^2}{b^2+\rho^2/4+\sqrt{D}} -\frac{1}{2}\ln\left[\frac{b^2+[1-2(1-2t)^2]\rho^2/4+\sqrt{D}}{2\rho^2\,t(1-t)}\right]\right\}, \quad \text{(B.23)}$$

and

$$D = \left(b^2 + \frac{\rho^2}{4}\right)^2 - b^2\rho^2(1-2t)^2 . \quad \text{(B.24)}$$

According to the above convention $\rho \ll b_0$ and this expression can be further simplified,

$$L(b_0,\rho,t) = \pi\rho^2(1-2t)\left\{1 - \ln\left[\frac{b_0^2}{\rho^2 t(1-t)}\right] + O\left(\frac{\rho^2}{b_0^2}\right)\right\} . \quad \text{(B.25)}$$

Now integration in (B.22) can be performed analytically, and we arrive at the final expression for σ_1

$$\sigma_1(\rho) = 4\pi(Z\alpha)^2\rho^2\left[\ln\left(\frac{b_0}{\rho}\right) - 2 - \mathrm{Re}\Psi(1+iZ\alpha) - C\right] , \quad \text{(B.26)}$$

where

$$\Psi(w) = \frac{\mathrm{d}}{\mathrm{d}\,w}\ln\Gamma(w) ; \quad \text{(B.27)}$$
$$C = -\Psi(1) = 0.5772 .$$

Adding eqs. (B.14) and (B.27), we eventually get the dipole cross-section (B.12) in the form

$$\sigma(\rho) = \sigma_{\mathrm{Born}}(\rho) - \Delta\sigma(\rho) . \quad \text{(B.28)}$$

Here

$$\sigma_{\mathrm{Born}}(\rho) = 4\pi(Z\alpha)^2\rho^2\left[\ln\left(\frac{a}{\rho}\right) - 2\right] ; \quad \text{(B.29)}$$

$$\Delta\sigma(\rho) = 4\pi(Z\alpha)^2\rho^2\left[\mathrm{Re}\Psi(1+iZ\alpha) + C\right] = 4\pi(Z\alpha)^2\rho^2 f(Z\alpha) , \quad \text{(B.30)}$$

and

$$f(Z\alpha) = (Z\alpha)^2\sum_{k=1}^{\infty}\frac{1}{k[k^2+(Z\alpha)^2]} . \quad \text{(B.31)}$$

Now we are in the position to calculate the photoproduction cross-section substituting eq. (B.28) into (1),

$$\sigma(\gamma Z \to e^+e^-Z) = \sigma_{\mathrm{Born}}(\gamma Z \to e^+e^-Z) - \Delta\sigma(\gamma Z \to e^+e^-Z) , \quad \text{(B.32)}$$

where

$$\sigma_{\mathrm{Born}}(\gamma Z \to e^+e^-Z) = \frac{28\,Z^2\alpha^3}{9\,m^2} \times \left[\ln(2am) - C - \frac{83}{42}\right] ; \quad \text{(B.33)}$$

$$\Delta\sigma(\gamma Z \to e^+e^-Z) = \frac{28\,Z^2\alpha^3}{9\,m^2} f(Z\alpha) . \quad \text{(B.34)}$$

Note that the Coulomb correction $\Delta\sigma(\gamma Z \to e^+e^-Z)$ in eq. (B.34) coincides with the one calculated in [21] using a very different technique. At the same time, the Born terms are different, as one should have expected. Indeed, the Bethe-Maximon theory describes photoproduction of e^+e^- pairs off a point-like nonscreened nucleus, while we are dealing with pair production off atoms. Equivalence of the Coulomb corrections calculated within the eikonal approximation and in the Bethe-Maximon theory in the case of photon bremsstrahlung can be proven exactly in the same way, since both processes are controlled by the same dipole cross-section.

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